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# Discussing the "nine chapters" of mathematics without the *Nine Chapters* in eighteenth-century Korea – the case of Cho T'ae-Gu

英家銘

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### Abstract

In this article, the author discusses a mathematical manuscript *Chusŏ kwangyŏn* (1718) written by a Korean noble scholar Cho T'ae-Gu (1660-1723). After reviewing the composition, contents and knowledge structure of the text, the author concludes that mathematicians of late-17th and early-18th centuries in Korea had a traditional East Asian knowledge structure of nine categories, although they did not have the entirety of the classical *Nine Chapters* at hand, to absorb and transform mathematics from China's Song-Yuan period and from the West. And Korean mathematicians in that period used freely what knowledge they had regardless of its origins to solve problems.

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## §1. Introduction

The *Nine Chapters of Mathematical Art* (*Jiuzhang suanshu* 九章算術, 1st century. From here on its title shall be abbreviated as "the *Nine Chapters*") was considered the most important mathematical classic in pre-modern East Asia. Ever since its introduction to the official examination systems in Tang, Silla and Japan in the 7th and 8th centuries, the text, along with its commentaries, had been seen as a "mathematical canon" (*suanjing* 算經) in East Asia. Its classification of mathematical knowledge, terminologies and form became standard for mathematical practices.[1] Although traditional East-Asian mathematics continued to develop afterwards, especially in the 11th to the 13th century when mathematicians presented many new methods in their works, much of the contents were based on the classification and terminologies in the *Nine Chapters*.[2]

Though being seen as a canon, the *Nine Chapters* was lost since the 15th century in both Ming China and Chosŏn Korea. Nevertheless, the term "nine chapters" and the nine categories of mathematical knowledge continued to be discussed in Chinese and Korean mathematical works, before the recovery of the original *Nine Chapters* and its commentaries in late-18th century in Qing China, and before its reintroduction to Chosŏn Korea possibly in the early 19th century.[3] For instance, in Ming China, Wu Jing 吳敬 wrote a book *Jiuzhang suanfa bilei daquan* 九章算法比類大全 (Great classified survey of the Nine Chapters, 1450) although they did not have the entirety of the *Nine Chapters* in the 15th century. In Chosŏn Korea, some mathematicians also talked about the nine categories in their works. The difference in Korea is that some of the mathematical methods that had been developed in the 13th century was lost in China but kept in Korea, such as *tianyuanshu* 天元術 (the method of the celestial source), so it is interesting to see whether or not those methods were classified into the nine categories.[4] Either way, it

[1] For the contents and influences of the *Nine Chapters*, refer to, for instance, Chemla, K. and Guo S., *Les neuf chapitres: Le classique mathematique de la Chine ancienne et ses commentaires*, Dunod, 2004. In particular, for introduction of the *Nine Chapters* in the official examination systems in East Asia in the 7th and 8th centuries, refer to 金容雲、金容局 『韓国数学史』、槙書店、1978。Also refer to 城地茂『和算の再発見：東洋で生まれたもう一つの数学』、化学同人、2014。

[2] For instance, Yang Hui 楊輝 presented new methods about root-extraction and equation-solving in his *Xiangjie Jiuzhang suanfa* 詳解九章算法 (Detailed explanations for the calculation methods in the Nine Chapters, 1262), the form, terminologies and classifications of which were based on the original *Nine Chapters* and its commentaries.

[3] On the recovery of the *Nine Chapters* in China, refer to, for example, Li, Y. and Du, S., *Chinese Mathematics: A Concise History*, Clarendon Press, 1987. The exact time of the reintroduction of the *Nine Chapters* to Korea is unclear, but it is no later than the mid-19th century. There is evidence that Korean scholar Kim Chŏng-Hŭi 金正喜 (1786-1856) possessed a newly printed version of the *Nine Chapters* by 1850s and later a mathematician Nam Pyŏng- Gil 南秉吉 (1820-1869) used that version to composed his own commentary, the *Kujang sulhae* 九章術解 (*Explanations of the mathematical methods of the Nine Chapters*, mid-19th century). Refer to 姜珉廷『『九章術解』의 연구와 역주』、成均館大學校博士論文、2015。

[4] The mathematical texts introducing *tianyunshu* were generally lost and recovered in the same time as the *Nine Chapters*. Refer to, for instance, 郭書春『中國科學技術典籍通彙·數學卷』第一分冊、河南教育出版社、1993，pp. 1046、1121.

would seem that the canon had been so influential in the past that scholars retained some parts of it in their collective memory during the time when it was still lost in the 17th and 18th centuries. So, how much did they really know about the original *Nine Chapters*, and what mathematical knowledge that was developed later was also classified into the nine categories? These are some of the intriguing questions that can be raised under this context. Moreover, Western mathematics was introduced into East Asia since the late 16th century, would there be any Western knowledge that was categorised into the "nine chapters"? The author wishes to explore the Korean side of the story. In fact, one Korean nobleman Cho T'ae-Gu 趙泰耇 (1660-1723) wrote a mathematical work *Chusŏ kwangyŏn* 籌書管見 (Humble view in a mathematical book, 1718), which contains sections specifically discussing what the author believed to be the contents of the nine categories. And the author of this paper intends to use this text to provide partial answers to those interesting questions mentioned above. In what follows, an account of Cho's life will be given, and then general contents of Cho's text shall be discussed.

## §2. An account of Cho T'ae-Gu's life

Cho T'ae-Gu was born in a noble *yangban* 兩班 family.[5] His father Cho Sa-Sŏk 趙師錫 (1632-93) was once the Right State Councillor (*Uŭijŏng* 右議政) of the Chosŏn court. Cho T'ae-Gu passed the official examination in 1686 when he was 26 years old. In 1710, he served as the Winter Solstice Chief Envoy (*Tongjisa* 冬至使) to Beijing. Between 1712 and 1714, he served as the Minister of Households. The Ministry of Households (*Hojo* 戶曹) was one of the organisations in the Chosŏn court that employed many middle-class (*chung'in* 中人) mathematical practitioners. During this period, Cho T'ae-Gu might have met and discussed mathematics with those middle-class officials, including one famous mathematician Hong Chŏng-Ha 洪正夏 (1684-?).[6] There is also record that Cho T'ae-Gu had contacts in 1713 with an astronomer Hŏ Won 許遠 of the State Observatory (*Kwansang-gam* 觀象監). Later in his life Cho was involved in serious power struggles. Although he became Chief State Councillor (*Yŏng'ŭijŏng* 領議政) in 1721-23, his political rivals took power after he died and deprived him of all his official positions posthumously. This is probably one reason that only one manuscript of his mathematical work survives to this day.[7]

The manuscript of Cho's mathematical work was finished in 1718, only a few years

5 The life story of Cho T'ae-Gu quoted in this paragraph mainly comes from the following works: 川原秀城〈東算と天元術 – 一七世紀中期～一八世紀初期の朝鮮数學〉、《朝鮮學報》169 (1998), 35-71。洪萬生〈朝鮮儒家讀九章 – 以趙泰耇〈九章問答〉為例〉、《國史館學術集刊》2 (2004), 297-324。吳建任『東算家趙泰耇《籌書管見》之內容分析》』、國立台灣師範大學數學系碩士論文、2007。

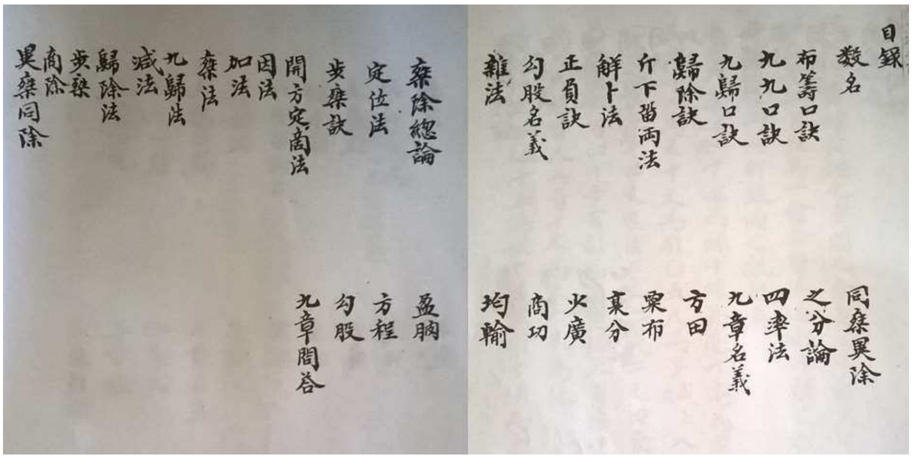
6 Hong S.S. and Hong Y.H., Chosun mathematics in the early 18th century, *Korean Journal for History of Mathematics* 25 (2012), 1-9.

7 A copy of the *Chusŏ kwangyŏn* is published in 金容雲『韓國科學技術史資料大系·數學卷（2）』、驪江出版社、1985， pp.1-199.

after he left his office in the Ministry of Households. Perhaps it was his contacts with those middle-class mathematical and astronomical practitioners in the Ministry of Households and in the State Observatory that prompted him to write his own mathematical ideas during and/or right after his services as the Minister of Households. Cho's work, a least from the term "humble view" in the title, suggests that the work contains his own thoughts and reflexions. In the next section the contents and possible sources of his mathematical ideas shall be discussed.

## §3. Contents of Cho's work and possible sources of his mathematical knowledge

Cho T'ae-Gu's mathematical work, the *Chusŏ kwangyŏn*, received quite different comments by modern scholars. For instance, [川原秀城, 1998] claims that the text is one of the mathematical works in the middle of the Chosŏn Dynasty that "maintain original plans to organise traditional East Asian mathematics, and to investigate its argumentative structures…".[8] However, [Hong and Hong, 2012] argues that the text is "strongly tied with Western mathematics".[9] Why are there such different opinions about the same text? The author believes that they are not mutually exclusive. Each presented some feature of this interesting text. We need to take a closer look into its contents and knowledge structure to understand this.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九章問答

Figure 1. List of contents of the *Chusŏ kwangyŏn*.[10]

Figure 1 is a list of contents of in the beginning of the *Chusŏ kwangyŏn*. As the reader can see, there are 37 sections in the work. The first 26 sections, from **Section 1**

[8] 『独自の構想をもって東アジアの傳統数学を整理し，その論理構造を究明する…』，川原秀城、1998, op. cit..

[9] Hong S.S. and Hong Y.H., 2012, op. cit.

[10] 金容雲、1985，op. cit., pp.3-4.

**Position Names/Units for Numbers** (*sumyŏng* 數名) to **Section 26 Method of the Four Proportional Terms** (*sayulbŏp* 四率法), can readily be recognised as mainly "basic rules for arithmetic and calculations", which in fact is quite similar to the contents and order of the *Suanxue qimeng zongua* 算學啟蒙總括 (Abstract of the Introduction to Mathematical Study), the opening section of the famous 13th-century Chinese text *Suanxue qimeng* 算學啟蒙 (Introduction to Mathematical Study) by Zhu Shijie 朱世傑.[11]

From the 27th section on, Cho T'ae-Gu begins to write about the "nine chapters", which will be discussed later. Among the first 26 sections of arithmetic rules, **Section 8 Positive and Negative Mnemonics** (*Chŏngbu'gyŏl* 正負訣) stands out. It is a section about the rules for addition and subtraction of positive and negative numbers, with only 40 characters:

> 其同名相減，則異名相加。正無人負之，負無人正之。
> 其異名相減，則同名相加。正無人正之，負無人負之。[12]

In the *Suanxue qimeng zonggua*, there, too, is this set of mnemonics, so this is also a clear sign that Cho's work was influenced by the *Suanxue qimeng*.

The first 26 sections constitute roughly one quarter of the whole text. Some of the sections in fact have only a few lines of simple rules such as the 8th section that is mentioned in the previous paragraph. From the 27th section, the author discusses directly what he believes to be the contents of the "nine chapters". The 27th section is about the "definitions" (*myŏng'ŭi* 名義) of the titles of the nine chapters, and then in the next nine sections, from the 28th to the 36th, the author discusses the contents for the nine categories.

**Section 27 Definitions of Nine Chapters** consists of only one paragraph with nine sentences, each of which is in fact closely parallel to the explanation by earlier Chinese commentators. For instance, its first sentence is "一曰方田，以御田疇界域" (The first is called "Rectangular Field", to handle the area of cultivated fields), which is identical to Li Hui's commentary in the 3rd century on the title of the first chapter of the *Nine Chapters*.[13] It would seem, at least from the "definition", that the range of the nine categories of mathematics in 18th-century Korea should match the traditional demarcations in ancient and medieval East Asia.

Now let us turn to the contents of the "nine chapters" in Cho's work. In what follows I shall give short descriptions of the titles and contents of the 28th to 36th sections, and compare them with the generally accepted contents of the original *Nine Chapters*.

**Section 28. Rectangular Field, Number One** (方田第一).[14] This section discusses

[11] For Zhu Shijie's work, see 郭書春、1993，op. cit., pp. 1119-1200.
[12] 金容雲、1985，op. cit., p.11.
[13] 金容雲、1985，op. cit., p.43; 郭書春、1993，op. cit., p.98.
[14] 金容雲、1985，op. cit., pp.43-51; 郭書春、1993，op. cit., pp.98-112.

the area of different planar shapes, including the circle. The original contents of the first chapter in the *Nine Chapters* actually begins with the arithmetic of fractions and related topics (such as the so-called "Euclidean algorithm" of finding greatest common divisors, for simplifying fractions) before going into areas of planar shapes. Cho covers fraction arithmetic earlier in the 25th section, in which he not only discusses the procedures for the four operations and the Euclidean algorithm but also explains how one can compare two fractions.[15] Cho did give several problems and solutions with fractional values in the dimensions of the shapes after he finished discussing the area procedures of the shapes in the current section, but he does not repeat the discussion about fractions. The shapes Cho discusses in this section are generally the same as those in the original *Nine Chapters*. He gives illustrations to each shape. For rectilinear shapes, he also gives graphical explanations of the area procedures (See Figure 2 for instances.). As the reader can see, the black regions in the figure are used to represent "void", the parts to be taken away or mended when calculating. This practice is most likely due to an influence from a 13th-century text widely circulated in Chosŏn Korea, the *Yang Hui suanfa* 楊輝算法 (Yang Hui's mathematical methods, 1274).[16]


Figure 2. Two diagrams explaining to area of triangles.[17]

**Section 29 Millet and Cloth, Number Two** (粟布第二).[18] This section deals with the exchange of commodities at different rates, and the problems of pricing, which are generally the same as those problems in the same chapter of the *Nine Chapters*. The title here is "Millet and Cloth", while in the original *Nine Chapters* it is "Millet and Rice" (*sumi* 粟米). Both texts discuss not only the exchange of grains but also of different textile, but maybe Cho wants to emphasise that the section does not only discuss grain exchanges. There are also problems of tax calculation according to land area, and those of interests. These seem to be problem of more practical concerns.

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15 金容雲、1985，op. cit., pp.33-39.

16 For a discussion of geometrical diagrams in traditional Korean mathematics, refer to Ying, J.M., A survey of geometrical diagrams in Korean mathematical texts from the 17th to the 19th century, *Historia Scientiarum* 23 (2013), 38-58.

17 金容雲、1985，op. cit., p.45.

18 金容雲、1985，op. cit., pp.51-59；郭書春、1993，op. cit., pp.112-120.

**Section 30 Weighted Distribution, Number Three** (衰分第三).[19] This section, as the corresponding chapter of the *Nine Chapters*, deals with problems of proportional distributions of commodities and currencies. One peculiarity is that in Cho's work there is also the problem of *jitu gongzu* 雞兔共足 (chickens, rabbits, and the sum of their legs), which has been famous in East Asia until the present day. The problem can be traced back to the *Sun Zi suanjing* 孫子算經 (Sun Zi's mathematical canon) in early imperial China, but not to the *Nine Chapters*.[20] It seems that Cho considers all problems about "uneven distribution" to be part of this category, regardless of its method of solutions. He actually put in this section several problems similar to some in the sixth chapter of the *Nine Chapters*, such as a problem whose answer is an arithmetic sequence. The common feature of these problems is that there are a few kinds of similar things (two kinds of animals, two kinds of textile, several sections of a bamboo…) and a shared amount of some quality (total legs of animals, total prices of textile, total volume of the bamboo sections…), and one has to distribute the quality unevenly according to some conditions.

**Section 31 Short Width, Number Four** (少廣第四).[21] Under the title of this section, Cho writes "[It is] just the method of root extraction" (即開方法), suggesting his own definition for it. In the original *Nine Chapters*, the first half of the fourth chapter is related to divisions by mixed fractions and common multiples, while the second half is about root extractions. As for this section in Cho's work, there is no problem of divisions of mixed fractions or common multiples, but the range of problems about root extractions are wider. They are about square and cube root extractions, finding sides given area and other conditions of a rectangle or volume of a cuboid with generalised root extraction procedures, and finding diameters given the area of a circle or volume of a sphere.

**Section 32 Consultation of Construction, Number Five** (商工第五).[22] This section deals with problems of volumes of solids and those of engineering, similar to its counterpart in the original *Nine Chapters*. However, two key solids, *yangma* 陽馬 and *bienao* 鼈臑, are missing.[23] Moreover, stacking methods (*t'oejŏkpŏp* 堆積法), which were sometimes called *duoji* 垛積 or *xiji* 隙積 in Chinese texts since the 13th century on, are included in this section. These problems are similar to those in the section *Duiji huanyuan men* 堆積還源門 in Zhu Shijie's *Sunaxue qimeng*.[24]

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19 金容雲、1985, op. cit., pp.59-66; 郭書春、1993, op. cit., pp.121-126.

20 For the *jitu gongzu* problem in the *Sun Zi suanjing*, see 郭書春、1993, op. cit., p.244.

21 金容雲、1985, op. cit., pp.66-83; 郭書春、1993, op. cit., pp.127-138.

22 金容雲、1985, op. cit., pp.83-92; 郭書春、1993, op. cit., pp.139-151.

23 A *yangma* is a pyramid with rectangular base and one lateral edge perpendicular to the base; a *bienao* is another kind of pyramid with right-triangular base and one lateral edge (but not that at the right-angled vertex of the base) perpendicular to the base. Liu Hui showed that the volume formulae for these two solids in the *Nine Chapters* were correct, and he used *yangma* as one of the basic solids to discuss the volumes of other solids in the fifth chapter. For Liu Hui's arguments, see Wagner, D. B., An early Chinese derivation of the volume of a pyramid, *Historia Mathematica*, 1979, 164-188.

24 郭書春、1993, op. cit., pp. 1172-1174.

**Section 33 Equitable Transportation, Number Six** (均輸第六).[25] This section has only seven problems. The first one is about the distribution of grains according to the population sizes and distances of three towns. The rest are mostly about more advanced problems of proportions. These are similar to the contents of the sixth chapter of the *Nine Chapters*. There is also a problem of least common multiples. It is unclear why Cho put this problem here.

**Section 34 Excess and Deficit, Number Seven** (盈朒第七).[26] This section of excess and deficit uses the method of false positions to solve problems, which is also similar to its counterpart in the original *Nine Chapters*.

**Section 35 Rectangular Array, Number Eight** (方程第八).[27] This section is about solving systems of linear equations, which is also similar to its counterpart in the original *Nine Chapters*.

**Section 36 Base and Height, Number Night** (句股第九).[28] This section, as suggested by its title, discusses problems about right-angled triangles. Beside traditional problems similar to those in the final chapter of the *Nine Chapters*, it also includes the famous measurement problem of "looking afar to the sea island" (*wanghaidao* 望海島) first given by Liu Hui in the 3rd century.[29]

The final section of the text, **Section 37 Questions and Answers about the Nine Chapters** (九章問答), is Cho T'ae-Gu's own explanations for the solution procedures of the problems in his nine categories of mathematics.[30] It takes up about one-third of the whole book. There are many interesting pieces of discussions in his nine sections for the nine categories and his explanations in the final section that we can take to see the possible sources of knowledge and his thoughts about the study of mathematics.

In the discussions for sections **28** to **36**, we have already seen that, besides contents that are similar to the original *Nine Chapters*, and those by Liu Hui and Sun Zi, there are problems and methods from medieval Chinese mathematical texts. This is by no means surprising, since earlier in the Chosŏn Dynasty in the 15th century, three Chinese texts from the Song (960-1279) and Yuan (1279-1368) periods were assigned as the subjects for examinations of mathematical officials. The *Yang Hui suanfa* (1274) and the *Suanxue qimeng* (1299) have already been mentioned, and the third one is the *Xiangming suanfa* 詳明算法(*Detailed and Explicit Mathematical Methods*, 1373).[31]

Other than sources of early imperial and medieval Chinese mathematics, there are indeed obvious traces of Western influences. The first clear instance is Cho's explanations

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25 金容雲、1985， op. cit., pp.92-95; 郭書春、1993， op. cit., pp.152-168.
26 金容雲、1985， op. cit., pp.95-101; 郭書春、1993， op. cit., pp.169-177.
27 金容雲、1985， op. cit., pp.101-112; 郭書春、1993， op. cit., pp.178-189.
28 金容雲、1985， op. cit., pp.112-120; 郭書春、1993， op. cit., pp.189-202.
29 For Liu Hui's measurement problems, see 郭書春、1993， op. cit., pp.217-224.
30 金容雲、1985， op. cit., pp.125-197. [洪萬生、2004] actually gives an interesting discussion about Cho's explanations to the procedures in this section.
31 金容雲、金容局『韓国数学史』、槙書店、1978, p.130.

for the procedure of the “looking afar to the sea island” problem. In the final section, Cho uses the concept of “similar shapes” (相似之形) to argue for the validity of the procedure.[32] However, it is well documented that similarity is not a Chinese concept of geometry.[33] Liu Hui used the “out-in complimentary principle” (*churu xiangbu* 出入相補) to show the procedure of the sea island problem was correct.[34] The concept of similarity was transmitted to China and later to Korea by the Chinese translation of Euclid’s *Elements*.[35] This idea is a clear instance that Cho was indeed influenced by Western mathematics.

Another trace of Western influence Cho received can be seen in his discussion of the method of the four *lü* in **Section 26**.[36] The simple proportional procedure (of finding the fourth term given three of the four terms when $a$: $b$ = $c$: $d$) is called *jinyou shu* 今有術 in the original *Nine Chapters*, but since the early 17th century the method had been called *silüfa* 四率法 in the Chinese works about Western astronomy and mathematics. So, using these terms can also be seen as a trace of Western influence on Cho’s work.

In fact, Cho specifically mentions “Westerners” (西士) in his work. In the second paragraph of the final section, he explains the mixture of units for length, area and volume. It is well-known that in traditional East-Asian mathematics, the names of the units for length, area and volume are the same.[37] In that paragraph, Cho explains the differences among dimensions and their units. After his explanation about length and units, he says “This is what the Westerners call the line” (此即西士所謂線也), and after those about area and units, he says “[This] is what the Westerners call the face” (即西士所謂面也).[38]

Cho’s drawing of geometrical diagrams also reveals his reception of Western influences. In the next section I shall discuss the geometrical diagrams in his work.

## §4. Cho’s use of geometrical diagrams

Cho T’ae-Gu uses a large number of diagrams in his work. Even if one does not consider the rod-counting expressions as “diagrams”, there are still more than 60 geometrical diagrams in his work. The main purpose of these diagrams, according to Cho himself, is to help the explanation of problems and procedures. He often says words such as the following when he uses diagrams: “This is difficult to explain with words, but it can be clarified with a diagram” (此難用言喩，可以圖明之).[39] Some of the diagrams

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32 See 金容雲、1985，op. cit., pp.185-188.
33 For instance, see Cullen, C., *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing*, Cambridge University Press, 1996, pp. 77-80.
34 郭書春『古代世界數學泰斗劉徽』、九章出版社、1995，p.200.
35 See 郭書春『中國科學技術典籍通彙·數學卷』第五分冊、河南教育出版社、1993，p.1261.
36 金容雲、1985，op. cit., pp.39-43.
37 王榮彬、李繼閔〈中國古代面積、體積度量制度考〉、《漢學研究》13 (2) (1995), 159-167。
38 金容雲、1985，op. cit., p. 127.
39 Hong S.S. and Hong Y.H., 2012, op. cit., p.8.

are simply representations of the geometrical objects in the problems they are attached to, such as the planar shapes in the section of Rectangular Field, or the right-angled triangles in the section of Base and Height. However, some diagrams are clearly intended to "explain" the truth of the procedures for calculation, as the reader has already seen in Figure 2.

The diagrams in Cho's texts have features from two traditions, European and East Asian. As many scholars have readily noticed, the diagrams in traditional East Asian mathematical texts were usually focused on the *transformations* of the general patterns of the objects rather than the structure of the geometrical objects themselves, which is the focus of diagrams in Euclidean geometry from Europe, and usually each vertex in a diagram in an European text is labelled with a letter to help describe different parts of the mathematical object in the argumentation of the desired results.[40] In what follows I shall show Cho's uses of East Asian and European diagrams.

**4.1 Diagrams with East-Asian features.**

Cho T'ae-Gu frequently uses East Asian style diagrams when showing or explaining procedures for finding areas of planar shapes. Beside Figure 2, there are several other diagrams that explain area procedures with the idea of transforming the original object (such as a trapezoid) into another object (such as a parallelogram), so one can easily find its area (See Figure 3). As mentioned before, black parts are the "void" to be mended or taken away. This is a prominent feature of East Asian style diagrams, and it is likely that Cho was influenced by Yang Hui for drawing them.

[40] For features of traditional East-Asian diagrams, see, for instance, Volkov, A., Geometrical diagrams in traditional Chinese mathematics, in F. Bray, V. Dorofeeva-Lichtmann and G. Métailié (eds.), *Graphics and Text in the Production of Technical Knowledge in China*, Brill, 2007, pp. 425-459. For a detailed discussion of Euclidean diagrams, see Netz, R., *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*, Cambridge University Press, 1999.

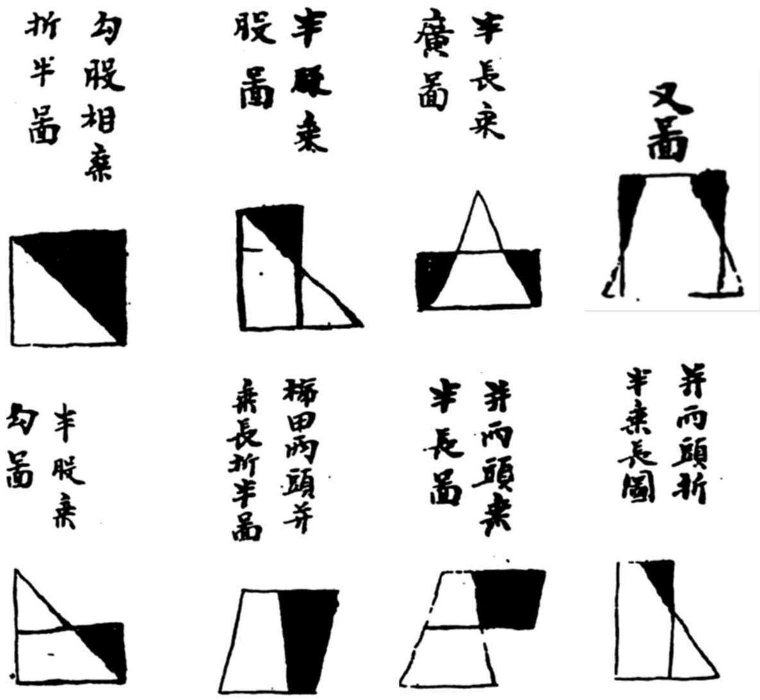

Figure 3. Diagrams explaining areas of planar figures.[41]

When explaining square root extractions, Cho also uses some East Asian style diagrams, in which the rectangular blocks, instead of the vertices, are labelled (See Figure 4).


Figure 4. “Diagram for square root extraction” (開平方圖).[42]

**4.2 Diagrams with European features.**

East Asian style diagrams are not the only kind of diagrams Cho uses in his work. When necessary, he uses European style diagrams. For example, again as mentioned

[41] 金容雲、1985，op. cit., pp.45-46.
[42] 金容雲、1985，op. cit., p150.

earlier, he uses "similar shapes", which is a Western idea, to argue for the validity of the procedure of "looking afar to the sea island" problem. The diagram attached to the Western method is a Euclidean style diagram in which the vertices are labelled, and it is used as a static diagram which suggests no transformation (See Figure 5).

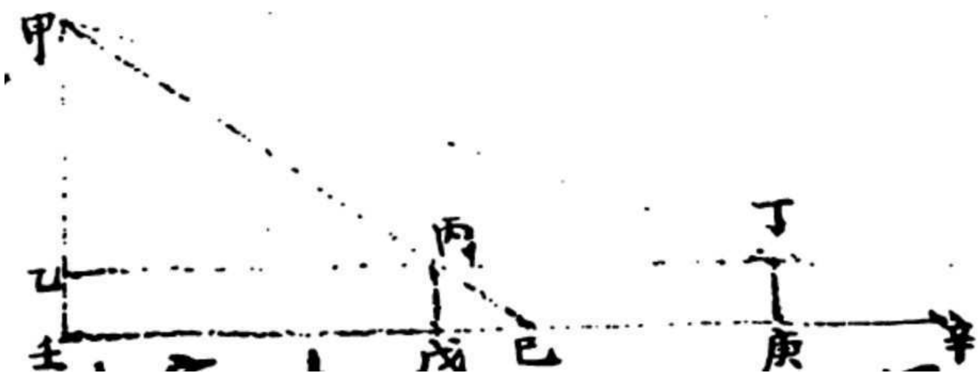

Figure 5. Cho's Diagram for the problem "looking afar to the sea island".[43]

Cho uses several other Euclidean style diagrams in his work when necessary, including two that represent three-dimensional solids. The representation of three-dimensional objects is also an interesting feature in Cho's work.

**4.3 Diagrams representing three-dimensional objects**

There are only three instances in which Cho draws representations of three-dimensional solids in his work. Two are related to volume problems, and the diagrams are obvious of Euclidean style (Figure 6).

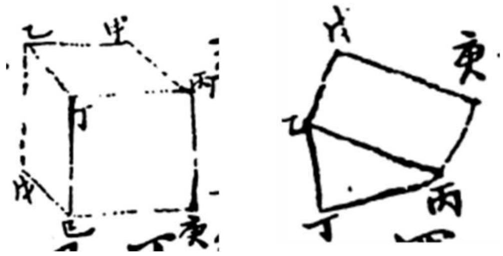
Figure 6. Diagrams for volume problems.[44]

Beside these two diagrams, there is a set of diagrams used to explain the procedure for cube root extractions (Figure 7).

[43] 金容雲、1985, op. cit., p. 186.
[44] 金容雲、1985, op. cit., pp. 165-166.

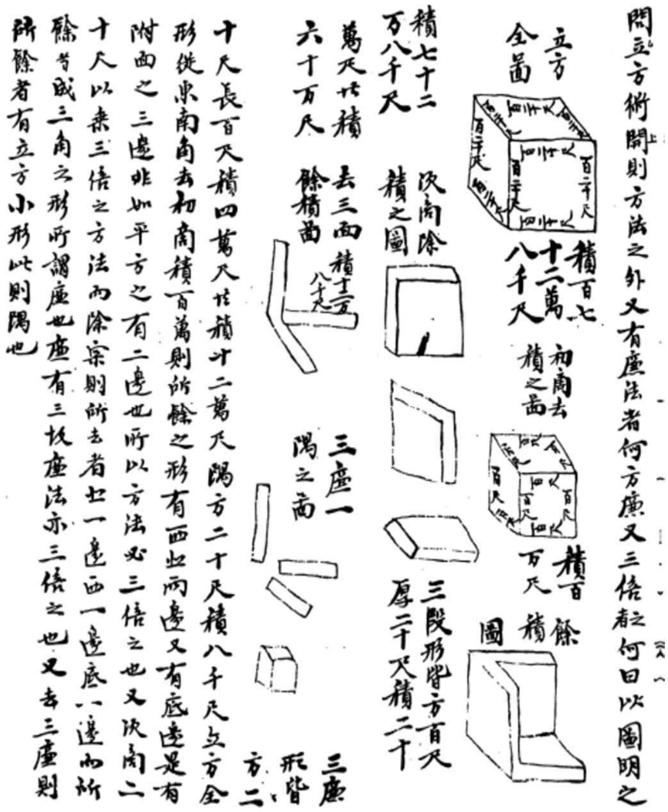

Figure 7. A set of diagrams attached to the explanation for cube root extractions.[45]

This set of diagrams is interesting for several reasons. First, the diagrams are not of Euclidean style. In fact, the edges of the two cubes are labelled, which is similar to East Asian style diagrams in which the sides of planar shapes are labelled. Second, among the Song-Yuan mathematical texts that were influential in Chosŏn Korea, none of them contains representations of three-dimensional solids. Third, in the few mathematical texts of 17th and early 18th centuries originally written by Korean mathematicians before Cho T’ae-Gu’s work, there is no representation of three-dimensional solids, either. It is not impossible that Cho was affected by some other Chinese or Korean mathematical works, but he was certainly one of the first Korean mathematicians who used representations of three-dimensional objects to help his explanations of procedures.

### §5. Mathematical knowledge structure in 18th-century Korea as seen through Cho T’ae-Gu’s work

Near the end of **Section 37**, which mainly contains Cho’s own explanations for the procedures in the nine categories, there is a long paragraph that shows clearly how Cho viewed the structure of mathematical studies:

[45] 金容雲、1985， op. cit., pp. 151-152.

> 或曰：數之為術，九章盡之矣！是外故無餘法乎？曰：…朱氏有立天元之法，西人有口平三角、弧三角之法，皆刱智而得其巧者也。[46] 然天元者，小廣之演也。三角，句股之奧也。豈能盡舍九章而為法哉！…其法理深奧，為術多方，有未易驟語，而領悟者，必須九章貫通無疑，然後可以近於此矣！[47]
> Someone says: For numbers to be [mathematical] procedures, the "nine chapters" have exhausted them. So is there not any remaining method outside [the nine chapters]? [I] say: […] Zhu [Shijie] had the method of setting up one celestial source, and the Westerners have those of planar and spherical trigonometry. Both have been creative with their wisdom and obtained the cleverness of their methods. However, what is called the celestial source is the development of "Short Width", and trigonometry is the depth of "Base and Height".[48] How can one abandon all of the nine chapters and use those methods! [...] The methods and principles [of the celestial source and trigonometry] are deep, their procedures are diversified, and it is not easy to give sudden discussions. And for those who want to understand, they must have a comprehensive knowledge of the nine chapters without any doubt. And then they can be close to these [advanced methods].

This paragraph, along with the contents in Cho's work, gives us a glance of how late-17th- and early-18th-century mathematicians viewed mathematics. Although the entirety of the original *Nine Chapters* was not available in their time, it would seem that they still divided the whole body of mathematical studies into nine categories. They do not merely discuss the ancient text *Nine Chapters*, but the "nine categories" of mathematics that seemed to include everything.[49] Cho does recognise in this paragraph that Chinese mathematician such as Zhu Shijie and the Westerners have their creative contributions, but their creations can still be included in the nine categories. The *tianyuanshu* was considered the extension of methods in the category of "Short Width", while trigonometry should be included in "Base and Height". Sections **28** to **36** about the contents of the nine categories tell the same story. The famous problem of chickens and rabbits belongs to the chapter of "Weighted Distribution", and the stacking problems in the Song-Yuan texts are part of the category of "Consultation of Construction". The nine categories are the entirety of mathematics.

## §6. Concluding remarks

[46] The box symbol "□" in the sentence represents a character that does not seem to fit into the context. It could have been a mistake by the copyist of the manuscript. Therefore, I put a box symbol and did not translate it.
[47] 金容雲、1985，op. cit., pp.193-194。
[48] In the original manuscript, it uses the term "小廣", which might be a synonym of "少廣" or just a mistake by the copyist. So I still translate the term as "Short Width", the title of the four chapter of the *Nine Chapters*.
[49] The idea of discussing "nine categories" instead of the ancient classic *Nine Chapters* was in fact inspired by Dr Catherine Jami of CNRS after my presentation of this paper in the conference in Kyoto University.

After reviewing the contents of the text *Chusŏ kwangyŏn* by Cho T’ae-Gu, a Korean noble mathematician of late-17th and early-18th centuries, some points can be concluded from discussions of the previous sections. Although it is certainly true that Cho and his text were influenced by Western mathematics, as shown by his use of methods such as similar shapes or of a few static diagrams with labelled vertices, the composition of the text and its knowledge structure is essentially East Asian. The most important sources of influence are still Song-Yuan texts such as the *Suanxue qimeng* and the *Yang Hui suanfa*. For the composition of Cho’s text, not only is the beginning quarter of the text modelled from a section of the *Suanxue qimeng*, mathematical knowledge and methods of that two Song-Yuan texts can be found throughout the whole of his work. As for the knowledge structure, it can also be seen that the nine categories of mathematics of the ancient class *Nine Chapters* are used to categorise the entirety of mathematics, including Western trigonometry and *tianyuanshu* of medieval China. Among this classical East Asian structure of mathematical knowledge, Cho uses freely whatever mathematics – whether they are classical from the original *Nine Chapters*, medieval from Song-Yuan periods, or of Western origins – that he sees fit to explain methods and solve problems in his work.

Cho T’ae-Gu’s work shows us the status of mathematical studies in the middle to late Chosŏn period, before the introduction to Korea of the overwhelmingly influential imperial canon of Qing, the *Shuli jingyun* 數理精蘊 (Essentials of Mathematical Principles, 1723).[50] Mathematicians maintained a traditional East Asian knowledge structure of nine categories, without the entirety of the *Nine Chapters* at hand, to absorb and transform into a coherent body medieval Chinese and early-modern Western mathematics, and they used freely what knowledge they have to solve practical or theoretical problems.

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